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Engineering Notes

Improved Iteration Algorithm for Nonlinear Vortex Lattice Method

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Nomenclature

U_∞	=	magnitude of free stream velocity
u, v, and w	=	induced velocity components at x, y, and z
ω	=	underrelaxation factor
x, y, and z	=	Cartesian coordinates of an endpoint of a vortex segment

Introduction

VORTEX lattice method (VLM) based on inviscid theory is widely used in practice as it is very effective for aerodynamic design. As conventional VLM is not suitable for aerodynamic analysis of low-aspect-ratio wings or wings at a high angle of attack, nonlinear VLM (NLVLM) was introduced in the 1970s [1–3]. Since then, NLVLM has been adopted frequently for aerodynamic analysis of low-aspect-ratio wings, delta wings, slender bodies, etc. (as can be found in [4–12]).

In NLVLM, vortex segments of a free vortex of a horseshoe vortex system are allowed to move to be aligned with local streamlines, and this inevitably requires an iterative numerical procedure. Convergence property of the iteration algorithm employed in the calculation is therefore a very practical concern [6,8,9]. In this work, we propose a modified iteration algorithm, which has been found to be more efficient and stable than the conventional method often adopted for NLVLM.

Improved Iteration Algorithm

NLVLM originates from VLM [10,11]. The main difference between the VLM and the NLVLM is the shape of the horseshoe vortex element. In the case of VLM, the shape of a trailing vortex is a semi-infinite straight line starting from the endpoint of a bound vortex to the downstream. On the contrary, the shape of the trailing vortex in NLVLM is curved so that it is aligned with the local streamline. To meet this requirement, each trailing vortex is broken into many vortex segments of finite length. The coordinates of vortex segments and the strength of the horseshoe vortices are to be determined simultaneously, and this requires an iterative solution procedure. Variables associated with vortex segments affect the convergence characteristics of NLVLM, so that studies on these variables concerning convergence have been performed to minimize the numerical instability of NLVLM [6,8,9].

A typical iteration procedure for NLVLM is illustrated in Fig. 1 [1]. In the process shown, the coordinates are updated by the following formula [7]:

$$\begin{aligned} y_j^{(i)} &= y_{j-1}^{(i)} + \frac{v_{j-1}^{(i)}}{U_\infty + u_{j-1}^{(i)}}(x_j - x_{j-1}) \\ z_j^{(i)} &= z_{j-1}^{(i)} + \frac{w_{j-1}^{(i)}}{U_\infty + u_{j-1}^{(i)}}(x_j - x_{j-1}) \end{aligned} \tag{1}$$

In Eq. (1), the superscript i denotes the iteration number, and the subscript j refers to the sequential number of the endpoints of a vortex segment. When we use Eq. (1) to determine a new position of a vortex segment, the iteration process often becomes unstable. Thus, for stability, we usually employ an underrelaxation scheme. That is, we use

$$X_k = \omega X_k^* + (1-\omega)X_{k-1} \tag{2}$$

where X_k^* is the vector $(y, z)^T$ updated by Eq. (1). Equations (1) and (2) compose the inner iteration loop of Fig. 1. The inner and outer iterations may diverge depending on the relaxation factor. Figure 2 gives an example. In Fig. 2, we plot the variation of C_L for a 75 deg delta wing at an angle of attack of 10 deg, with the inner iteration number for a given outer iteration cycle [13]. We clearly see that the iteration diverges when $\omega = 0.7$.

To our surprise, an alternative iteration algorithm to mitigate this undesirable instability has not been found from open literatures. We thus propose, in this work, an improved algorithm that is simple and stable, which is given by

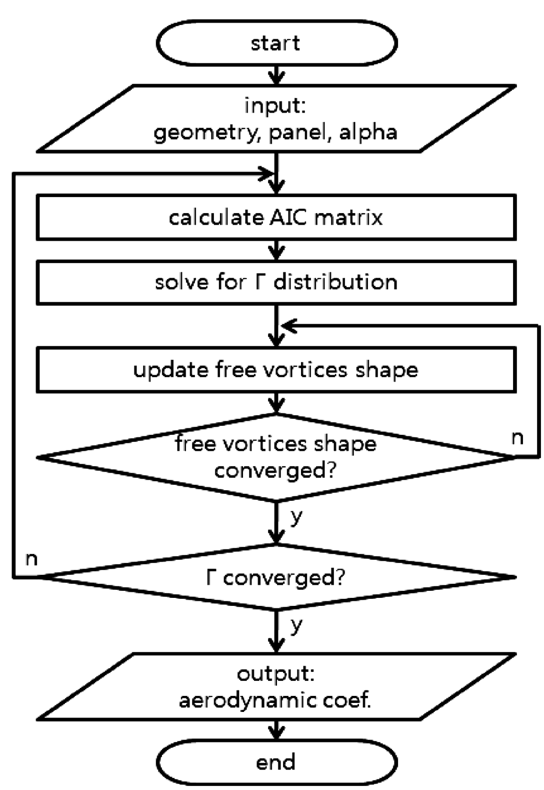

Fig. 1 Conventional NLVLM solution procedure.

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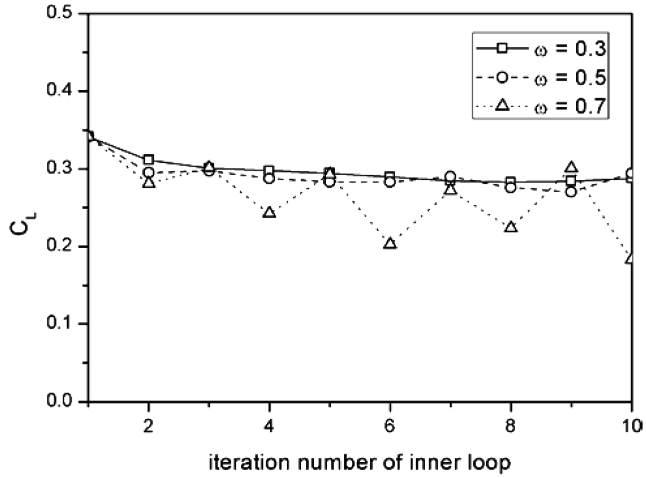

Fig. 2 Variation of C_L with the iteration number at various ω.

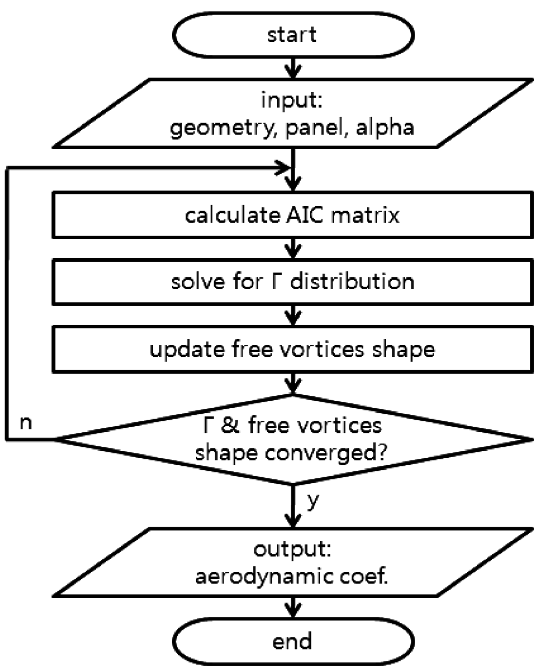

Fig. 3 Present NLVLM solution procedure.

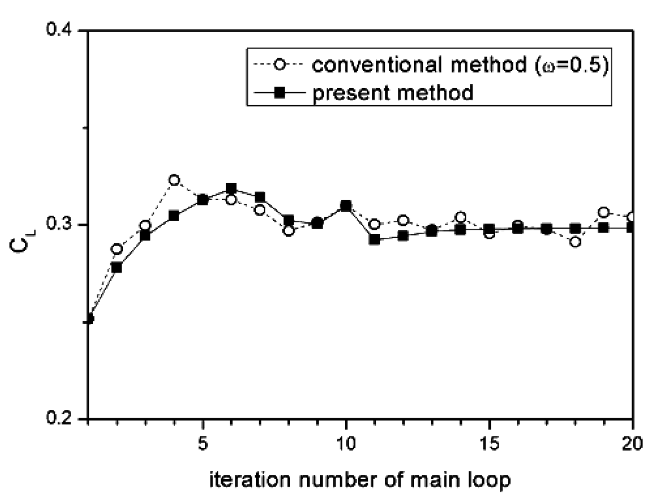

Fig. 4 Variation of C_L with the iteration number.

$$y_j^{(i)} = y_{j-1}^{(i-1)} + \frac{v_{j-1}^{(i)}}{U_\infty + u_{j-1}^{(i)}}(x_j - x_{j-1})$$

$$z_j^{(i)} = z_{j-1}^{(i-1)} + \frac{w_{j-1}^{(i)}}{U_\infty + u_{j-1}^{(i)}}(x_j - x_{j-1}) \tag{3}$$

As can easily be identified, the only difference between Eqs. (1) and (3) is the use of $y_{j-1}^{(i-1)}$ and $z_{j-1}^{(i-1)}$ in Eq. (3) instead of $y_{j-1}^{(i)}$ and $z_{j-1}^{(i)}$ in Eq. (1). The idea for this simple change is that the motion of the vortex segment of a trailing vortex may be simulated better if we mimic the shedding of the vortex line; this would be better represented by using $y_{j-1}^{(i-1)}$ and $z_{j-1}^{(i-1)}$, as these can be interpreted as previous time values. We suggest the overall solution procedure with Eq. (3), which is shown in Fig. 3. From Fig. 3, we find that the inner iteration loop (including the underrelaxation step of Fig. 1) is absent, which makes the solution process much simpler. Figure 4 compares the variations of C_L of the same wing of Fig. 2 with the iteration number for the two different cases. We clearly see that the present method exhibits a superior convergence property over the conventional procedure. We confirm that the present method also works very well for the cases of rectangular wings of aspect ratios of 1 and 2 and delta wings of aspect ratios ranging from 1 to 4 [13].

Conclusions

We proposed a new iterative algorithm for the NLVLM that was simpler and more stable. Performance of the proposed method was demonstrated through example calculations for the rectangular and delta wing with a low-aspect ratio.

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